

Descartes's Rule of Signs

Let $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$ be a polynomial with real coefficients.

1. The number of *positive real zeros* of f is either
 - a. the same as the number of sign changes of $f(x)$
 - or
 - b. less than the number of sign changes of $f(x)$ by a positive even integer.
If $f(x)$ has only one variation in sign, then f has exactly one positive real zero.
2. The number of *negative real zeros* of f is either
 - a. the same as the number of sign changes of $f(-x)$
 - or
 - b. less than the number of sign changes of $f(-x)$ by a positive even integer.
If $f(-x)$ has only one variation in sign, then f has exactly one negative real zero.

Example 5: Use Descartes's Rule of Signs to determine the possible number of positive and negative real zeros for each given function.