

The Fundamental Theorem of Algebra

If $f(x)$ is a polynomial of degree n , where $n \geq 1$, then the equation $f(x) = 0$ has at least one complex root.

The Linear Factorization Theorem

If $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$, where $n \geq 1$ and $a_n \neq 0$, then

$$f(x) = a_n(x - c_1)(x - c_2) \cdots (x - c_n),$$

where c_1, c_2, \dots, c_n are complex numbers (possibly real and not necessarily distinct).

In words: An n th-degree polynomial can be expressed as the product of a nonzero constant and n linear factors, where each linear factor has a leading coefficient of 1.

Example 4: Find an n th-degree polynomial function with real coefficients satisfying the given conditions. If you are using a graphing utility, use it to graph the function and verify the real zeros and the given function value.