

The Leading Coefficient Test

As x increases or decreases without bound, the graph of the polynomial function

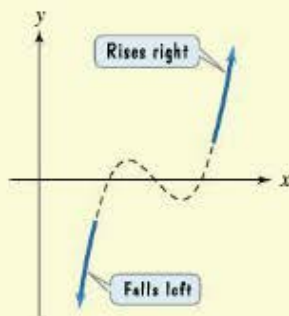
$$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_1 x + a_0 \quad (a_n \neq 0)$$

eventually rises or falls. In particular,

1. For n odd:

If the leading coefficient is positive, the graph falls to the left and rises to the right. (\swarrow , \nearrow)

$$a_n > 0$$



Odd degree; positive leading coefficient

If the leading coefficient is negative, the graph rises to the left and falls to the right. (\nwarrow , \searrow)

$$a_n < 0$$

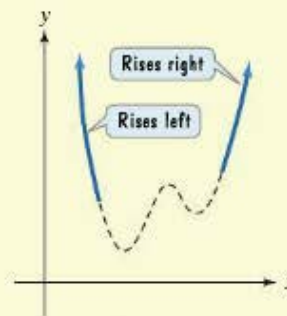


Odd degree; negative leading coefficient

2. For n even:

If the leading coefficient is positive, the graph rises to the left and rises to the right. (\nwarrow , \nearrow)

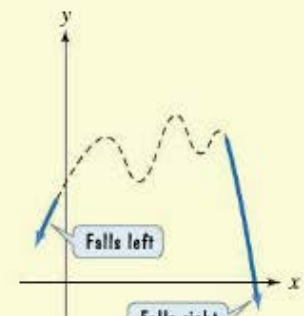
$$a_n > 0$$



Even degree; positive leading coefficient

If the leading coefficient is negative, the graph falls to the left and falls to the right. (\swarrow , \searrow)

$$a_n < 0$$



Even degree; negative leading coefficient

Odd-degree polynomial functions have graphs with opposite behavior at each end.

Even-degree polynomial functions have graphs with the same behavior at each end.

Example 2: Use the Leading Coefficient Test to determine the end behavior of the graph of the given polynomial function.