

The Algebra of Functions: Sum, Difference, Product, and Quotient of Functions

Let f and g be two functions. The **sum** $f + g$, the **difference** $f - g$, the **product** fg , and the **quotient** $\frac{f}{g}$

are functions whose domains are the set of all real numbers common to the domains of f and g ($D_f \cap D_g$). Defined as follows:

1. Sum: $(f + g)(x) = f(x) + g(x)$
2. Difference: $(f - g)(x) = f(x) - g(x)$
3. Product: $(fg)(x) = f(x) \cdot g(x)$
4. Quotient: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, provided $g(x) \neq 0$.

Example 2: Find $f + g$, $f - g$, fg , and $\frac{f}{g}$. Determine the domain for each function.