

Solving Radical Equations of the Form $x^{\frac{m}{n}} = k$

Assume that m and n are positive integers, $\frac{m}{n}$ is in lowest terms, and k is a real number.

1. Isolate the expression with the rational exponent.
2. Raise both sides of the equation $\frac{n}{m}$ power.

If m is even = k

$$\begin{aligned} x^{\frac{m}{n}} &= k \\ \left(x^{\frac{m}{n}}\right)^{\frac{n}{m}} &= \pm k^{\frac{n}{m}} \\ x &= \pm k^{\frac{n}{m}} \end{aligned}$$

If m is odd:

$$\begin{aligned} x^{\frac{m}{n}} &= k \\ \left(x^{\frac{m}{n}}\right)^{\frac{n}{m}} &= k^{\frac{n}{m}} \\ x &= k^{\frac{n}{m}} \end{aligned}$$

It is incorrect to insert the \pm symbol when the numerator of the exponent is odd. An odd index has only one root.

3. Check all proposed solutions in the original equation to find out if they are actual solutions or extraneous solutions.

Example 3: Solve each equation with rational exponents. Check all proposed solutions.